

NAG Toolbox for MATLAB

s17ae

1 Purpose

s17ae returns the value of the Bessel Function $J_0(x)$, via the function name.

2 Syntax

```
[result, ifail] = s17ae(x)
```

3 Description

s17ae evaluates an approximation to the Bessel Function of the first kind $J_0(x)$.

Note: $J_0(-x) = J_0(x)$, so the approximation need only consider $x \geq 0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 8$,

$$J_0(x) = \sum_{r=0}' a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$J_0(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_0(x) \cos\left(x - \frac{\pi}{4}\right) - Q_0(x) \sin\left(x - \frac{\pi}{4}\right) \right\},$$

where $P_0(x) = \sum_{r=0}' b_r T_r(t)$,

and $Q_0(x) = \frac{8}{x} \sum_{r=0}' c_r T_r(t)$,

with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $J_0(x) \simeq 1$. This approximation is used when x is sufficiently small for the result to be correct to **machine precision**.

For very large x , it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of

oscillation of $J_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi|x|}}$, can be determined and this is returned on soft failure. The range for which this occurs is roughly related to **machine precision**; the function will fail if $|x| \gtrsim 1/\text{machine precision}$.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W 1962 *Mathematical tables Chebyshev Series for Mathematical Functions* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result** – double scalar

The result of the function.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

x is too large. On soft failure the function returns the amplitude of the J_0 oscillation, $\sqrt{\frac{2}{\pi|x|}}$.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $J_0(x)$ oscillates about zero, absolute error and not relative error is significant.)

If δ is somewhat larger than the *machine precision* (e.g., if δ is due to data errors etc.), then E and δ are approximately related by:

$$E \simeq |xJ_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xJ_1(x)|$.

However, if δ is of the same order as *machine precision*, then rounding errors could make E slightly larger than the above relation predicts.

For very large x , the above relation ceases to apply. In this region, $J_0(x) \simeq \sqrt{\frac{2}{\pi|x|}} \cos\left(x - \frac{\pi}{4}\right)$. The amplitude $\sqrt{\frac{2}{\pi|x|}}$ can be calculated with reasonable accuracy for all x , but $\cos\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\cos\left(x - \frac{\pi}{4}\right)$ is determined by θ only. If $x \gtrsim \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of, the inverse of the *machine precision*, it is impossible to calculate the phase of $J_0(x)$ and the function must fail.

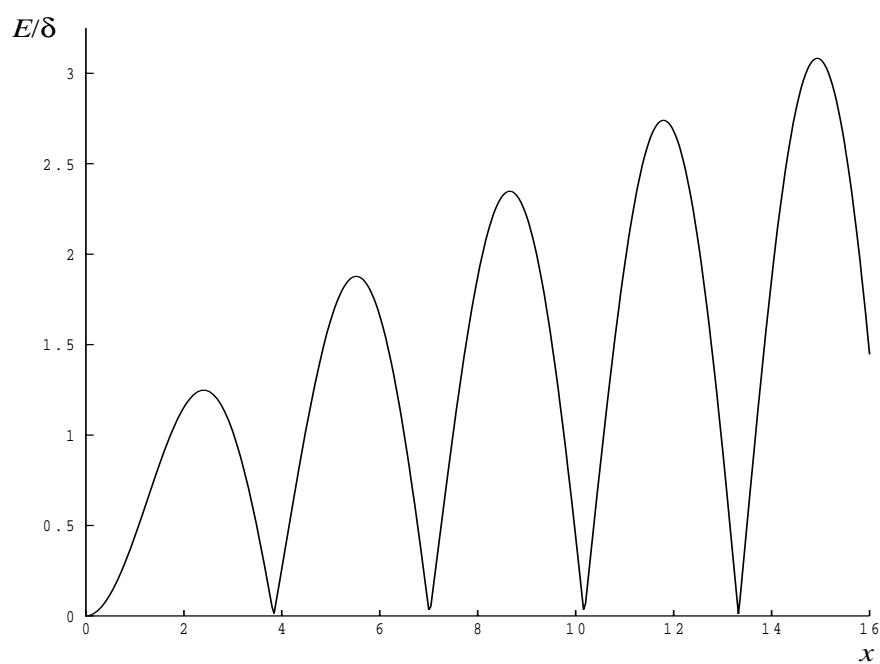


Figure 1

8 Further Comments

None.

9 Example

```
x = 0;  
[result, ifail] = s17ae(x)
```

```
result =  
    1  
ifail =  
    0
```